Relativistic form of Newton's law of gravitation.

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Newton's law of gravity can be modified so that it explicitly depends on the speed. The reasoning is as follows.

Let's say we have a gravitationally bound system of two elementary particles with masses M and m1 (M and m1 are the rest masses). Moreover, particle m1 moves with velocity v along a circular trajectory around particle M at a distance r.

The binding energy of the system, as usual, will follow from the mass defect: E = Δm * c^2. Where Δm is the relativistic increment of the mass of particle m1.

m = m1 / (1 - v^2/c^2)^0.5

Δm = m - m1

This means that the centripetal force (F = ma = m * v^2/r) of the moving particle will depend on the relativistic mass of the particle. In this case, we simply have to take into account the relativistic effects in Newton's law of gravity, and therefore, it is the relativistic mass of the moving particle that will be taken into account.

Since in the frame of reference associated with a stationary particle M, with uniform rectilinear motion of a particle with rest mass m1 with speed v tangent to the circle, at a point located on the circle, the observer will record a relativistic increase in the particle mass by Δm (in comparison with the rest mass m1; the observer will register the mass m). This is true for all points on the circle. That is, during circular motion, at each moment in time, for a given observer, a particle with mass M gravitationally interacts with a particle with mass m located at a distance r. This is why we need to take into account relativism in Newton's law of gravitation. Then the equation will be written as:

F = (G * M * m1) / (r^2 * (1 - v^2/c^2)^0.5)

M and m1 are the rest masses.

We take into account the expansion in the Maclaurin series and write down the first three terms (c is the speed of light).

F = (G * M * m1) / r^2 + (v^2 * G * M * m1) / (2 * c^2 * r^2) + (3 * v^4 * G * M * m1) / (8 * c^4 * r^2) +..

For simplicity, we leave two terms.

F = (G * M * m1) / r^2 + (v^2 * G * M * m1) / (2 * c^2 * r^2)

As we can see, an increase in the speed of a planet (Mercury) or satellite (Pioneer anomaly) will lead to a more significant deviation from the classical Newtonian law of gravity. This also explains the flyby anomaly. In the general case, it is necessary to take into account the "longitudinal" and "transverse" mass.

Finally, let me remind you that the speed of stars and galaxies is quite high compared to the orbital speed of the Earth (29.8 km/s). For example, the orbital speed of our Sun is 217 km/s, and the speed of the Milky Way relative to the nearest galaxies is estimated at about 600 km/s.

Appendix.

Given this modification of Newton's classical law of gravitation, we can write the correct relativistic form of the previously obtained generalized law of gravitation (based on the motion of stars in galaxies, and the recession of galaxies). The generalized form of the equation is [1]:

F = (K1 * M * m)/(r^2) + (K2 * M * m)/r + K3 * M * m

Then the relativistic form can be written as:

F = (K1 * M * m1)/(r^2 * (1 - v^2/c^2)^0.5) + (K2 * M * m1)/(r * (1 - v^2/c^2)^0.5) +
(K3 * M * m1)/(1 - v^2/c^2)^1.5

Of course, the masses M and m1 are the rest masses, and K1, K2 and K3 are constants.

For the second and third terms, it is precisely this relativistic form and this approach that we used earlier to explain the motion of stars in galaxies and to derive the Hubble-Lemaitre law [1, pp. 13 - 22].

The motion of planets and satellites in the Solar System, I think, is better described by the first two terms of the equation, especially at the outer edges of the Solar System:

F = (K1 * M * m1)/(r^2 * (1 - v^2/c^2)^0.5) + (K2 * M * m1)/(r * (1 - v^2/c^2)^0.5)

The motion of stars in galaxies is described by the second term:

F = (K2 * M * m1)/(r * (1 - v^2/c^2)^0.5)

The recession of galaxies is described by the third term:

F = (K3 * M * m1)/(1 - v^2/c^2)^1.5

1. Bezverkhniy V. D., Bezverkhniy V. V. Newton's Gravity Depending on the Topology of Space. SSRN Electronic Journal, Jul 2019. P. 8 - 10. https://dx.doi.org/10.2139/ssrn.3412216